Ex: ① $\int_0^1 \sqrt[3]{1+7x}\,dx$

$\left(\begin{array}{l} u = 1+7x \\ du = 7dx \rightarrow \frac{1}{7}du = dx \end{array} \right.$

$= \int_1^8 \sqrt[3]{u}\left(\frac{1}{7}du\right)$

$x=1 \longrightarrow u = 1+7\cdot 1 = 8$

$x=0 \longrightarrow u = 1+7\cdot 0 = 1$

$= \frac{1}{7}\int_1^8 u^{1/3}\,du = \frac{1}{7}\left(\frac{3}{4}u^{4/3}\right)\Big|_1^8$

$= \frac{3}{28}\left(8^{4/3} - 1^{4/3}\right) = \frac{3}{28}(16-1) = \frac{45}{28}$

② $\int_1^2 \frac{e^{1/x}}{x^2}\,dx$

$\left(\begin{array}{l} u = \frac{1}{x} \quad du = \frac{-1}{x^2}dx \\ x=1 \rightarrow u = \frac{1}{1} = 1 \\ x=2 \rightarrow u = \frac{1}{2} \end{array} \right)$

$= \int_1^{1/2} -e^u\,du$

$= -e^u\Big|_1^{1/2} = -e^{1/2} - (-e^1) = \boxed{e - e^{1/2}}$

Facts:

Suppose f is integrable, and F is an antiderivative of f.

① $$\int f(kx)\,dx = \frac{1}{k}F(kx) + C$$

e.g.: $$\int e^{2x}\,dx = \frac{1}{2}e^{2x} + C$$

$$\int \cos(4x)\,dx = \frac{1}{4}\sin(4x) + C$$

② $$\int f(kx+a)\,dx = \frac{1}{k}F(kx+a) + C$$

e.g.: $$\int \sin(\pi x - 4)\,dx = \frac{-1}{\pi}\cos(\pi x - 4) + C$$

Ex: $\int \ln x \, dx = x\ln x - x + C$

Ex: $\int x e^x \, dx$

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

$$\cancel{\int \frac{d}{dx}}\left[f(x)g(x)\right] \cancel{dx} = \int \left(f'(x)g(x) + f(x)g'(x)\right)dx$$

$$f(x)g(x) + C = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

Integration by Parts

$$\boxed{\int \underbrace{f(x)g'(x)}_{\text{product}}\,dx = f(x)g(x) - \int f'(x)g(x)\,dx}$$

Let $u = f(x)$ & $v = g(x)$. Then $du = f'(x)\,dx$ and $dv = g'(x)\,dx$, so we can rewrite <u>Integration by Parts</u> as:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Ex: $\int \overbrace{x e^x \, dx}^{u\,dv}$ $\left(\begin{array}{ll} u = x & dv = e^x dx \\ du = dx & v = e^x \end{array} \right)$

$$= \overbrace{x e^x}^{uv} - \int \overbrace{e^x \, dx}^{v\,du}$$

$$= x e^x - e^x + C$$

How to pick u & dv?

Pick u to be a complicated piece whose derivative becomes simpler.

Pick dv to be the most complicated thing we can integrate.

Choose u to be the piece of the integrand that shows up first in the acronym:

L ogarithmic

I nverse Trig

A lgebraic

T rig

E xponential

Once u is picked, dv is the remaining portion of the integrand.